Numerical Methods for Partial Differential Equations. An Introduction

Description: Numerical Methods for Partial Differential Equations: An Introduction

Vitoriano Ruas, Sorbonne Universités, UPMC – Université Paris 6, France

A comprehensive overview of techniques for the computational solution of PDE's

Numerical Methods for Partial Differential Equations: An Introduction covers the three most popular methods for solving partial differential equations: the finite difference method, the finite element method and the finite volume method. The book combines clear descriptions of the three methods, their reliability, and practical implementation aspects. Justifications for why numerical methods for the main classes of PDE's work or not, or how well they work, are supplied and exemplified.

Aimed primarily at students of Engineering, Mathematics, Computer Science, Physics and Chemistry among others this book offers a substantial insight into the principles numerical methods in this class of problems are based upon. The book can also be used as a reference for research work on numerical methods for PDEs.

Key features:

A balanced emphasis is given to both practical considerations and a rigorous mathematical treatment.

The reliability analyses for the three methods are carried out in a unified framework and in a structured and visible manner, for the basic types of PDE's.

Special attention is given to low order methods, as practitioner's overwhelming default options for everyday use.

New techniques are employed to derive known results, thereby simplifying their proof.

Supplementary material is available from a companion website.

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